

Zee-type Neutrino Mass Matrix and Bi-Maximal Mixing

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Abstract

It is shown that the bi-maximal solution is the only possibility to reconcile Zee-type neutrino mass matrix with three flavors to the current atmospheric and solar neutrino experimental data. The mass of the lightest neutrino, which consist mostly of ν_μ and ν_τ ,

is $\simeq \Delta m_\odot^2 / (2\sqrt{\Delta m_{atm}^2})$. The related topics on Zee-type neutrino mass matrix are also discussed.

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1 Introduction

Recent atmospheric[1] and solar[2] neutrino experiments suggest that neutrinos have tiny masses compared to quarks and charged leptons and the MNS mixing matrix is different from the CKM matrix in the quark sector. The tiny masses for neutrinos are expected to be less than $\mathcal{O}(1)\text{eV}$. It is well known that there are mainly two mechanisms to obtain tiny neutrino masses. One is the seesaw mechanism, which requires the existence of right-handed Majorana neutrinos with $10^{10} - 10^{15}\text{GeV}$ masses. The other mechanism is to generate tiny masses by higher order loop effects by the extension of the standard model(SM). In both scenarios light neutrinos are Majorana. In this talk[3] we investigate the second possibility to interpret the tiny neutrino masses and mixing patterns suggested by experiments[1, 2] within the framework of three flavors.

2 Zee Model

A model with extension of the Higgs sector by adding one more doublet in $SU(2)_L$ and a charged singlet scalar with lepton number 2 was proposed by Zee[4] almost two decades ago. The Zee model is the minimal extension of the SM and the one loop effects by extended Higgs sector can induce small Majorana neutrino masses. In the Zee model the neutrino masses are given by

$$m_{ll'} = f_{ll'}(m_l^2 - m_{l'}^2)\frac{\mu v_u}{v_d}F(M_1^2, M_2^2), \quad (1)$$

where $M_{1,2}$ are the masses of the two physical charged scalars. This type of neutrino matrix has the following characteristic features;

1. The diagonal elements are zero due to antisymmetric properties $f_{ll'} = -f_{l'l}$.
2. There is no CP violation because the three phases in the mass matrix are absorbed by the three Majorana neutrino fields. However, if we take into account of two loop effects there appear tiny non-zero masses at diagonal parts and it is possible to obtain CP violation in this model[5].
3. If we assume naively the relations $f_{e\mu} \simeq f_{e\tau} \simeq f_{\mu\tau}$, it is expected that the relation $m_{\mu\tau} \simeq m_{e\tau} \gg m_{e\mu}$ [6] due to $m_{ij} \propto f_{ij}(m_i^2 - m_j^2)$. However the mass and mixing matrices are inconsistent with the result of CHOOZ's experiments[7].

In this analysis we call the mass matrix given by Eq.(1) as Zee-type neutrino mass matrix and we investigate the consistent solution of Zee type mass matrix with experiments generally by relaxing the relation $f_{e\mu} \simeq f_{e\tau} \simeq f_{\mu\tau}$ [8].

3 Bi-maximal MNS Matrix

The mass matrix (1) is symmetric and diagonalization is done by the MNS mixing matrix

$$U_{MNS} = \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 \\ -s_1 c_2 - c_1 s_2 s_3 & c_1 c_2 - s_1 s_2 s_3 & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 & -c_1 s_2 - s_1 c_2 s_3 & c_2 c_3 \end{pmatrix}. \quad (2)$$

We obtain the following conditions due to the property of Zee-type mass matrix;

$$\begin{aligned} (1, 1) \quad & m_1 c_1^2 c_3^2 + m_2 s_1^2 c_3^2 + m_3 s_3^2 = 0, \\ (2, 2) \quad & m_1 (s_1 c_2 + c_1 s_2 s_3)^2 + m_2 (c_1 c_2 - s_1 s_2 s_3)^2 + m_3 s_2^2 c_3^2 = 0, \\ (3, 3) \quad & m_1 (s_1 s_2 - c_1 c_2 s_3)^2 + m_2 (c_1 s_2 + s_1 c_2 s_3)^2 + m_3 c_2^2 c_3^2 = 0, \end{aligned}$$

where $m_i (i = 1, 2, 3)$ is the eigenvalues of Eq.(1). Then the relations

$$\begin{aligned} m_2 &= -\frac{\cos^2 \theta_1 - \tan^2 \theta_3}{\sin^2 \theta_1 - \tan^2 \theta_3} m_1, & m_3 &= -m_1 - m_2 \\ \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 &= \frac{1}{2} \sin 2\theta_1 \sin 2\theta_2 (3 \cos^2 \theta_3 - 2) \sin \theta_3 \end{aligned} \quad (3)$$

are derived. It is noted that the hierarchical mass solution like $m_3 \gg m_2 \gg m_1$ is prohibited in this model by the traceless condition.

Here we input the requisite condition $\theta_2 \simeq \pi/4$ to obtain large mixing suggested by the atmospheric neutrino experiments[1]. From Eq.(3) the possible solutions are $\theta_3 \simeq 0$,

$\theta_1 \simeq 0$ or $\theta_3 \simeq \arctan \sqrt{1/2}$. However the latter two cases except for $\theta_1 \simeq \theta_3 \simeq 0$ require

$|U_{e3}| > 0.22$ and this solution is contradicting with the CHOOZ experiment[7]. Then it is enough to investigate the case of $\theta_3 \simeq 0$. There are two hierarchical mass squared differences $\Delta m_{atm}^2 \gg \Delta m_{solar}$ and in this model the unique solution is given for $\theta_3 \simeq 0$ in Eq.(2).

The degenerate solutions with two masses are given for (1) $\theta_1 \simeq 0$, (2) $\theta_1 \simeq \arctan 1/\sqrt{2}$ and (3) $\theta_1 \simeq \pi/4$. The former two cases are inconsistent with the experimental data[3]. The consistent solution is given only for the last case, which implies the solution $|m_1| \simeq |m_2| \gg |m_3|$, $\Delta m_{13}^2 \simeq \Delta m_{23}^2 = \Delta m_{atm}^2$, $\Delta m_{12}^2 = \Delta m_{\odot}^2$. The mixing matrix is obtained as

$$U^{Zee} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

This corresponds to the MSW or vacuum large angle solution in solar neutrino experiments. Only the third case ($\theta_1 \simeq \pi/4, \theta_2 \simeq \pi/4, \theta_3 \simeq 0$) can give a consistent solution and the MNS mixing matrix is bi-maximal. The neutrino masses are $|m_1| \simeq |m_2| \simeq$

$\sqrt{\Delta m_{atm}^2}$, $|m_3| \simeq \frac{\Delta m_{\odot}^2}{2\sqrt{\Delta m_{atm}^2}}$ and an anti-hierarchical mass structure is obtained. The re-

lation $m_1 \simeq -m_2$ implies pseudo-Dirac solution. For the parameters in the Zee model the consistent solution with experiments suggests a hierarchical relation as $f_{e\mu} \gg f_{e\tau} \gg f_{\mu\tau}$.

4 Summary

The unique solution of the model is the bi-maximal one, which corresponds to MSW or vacuum large mixing angle solution in solar neutrino oscillation. The neutrino masses are

$|m_1| \simeq |m_2| \simeq 0.02 \sim 0.08\text{eV} (|m_3| \ll |m_1| \simeq |m_2|)$. The neutrinoless double β decay is induced by the (1,1) element in the mass matrix and $\langle m_\nu \rangle = |\sum_i U_{ei}^2 m_{\nu i}| = |(1,1)| \approx 0$ prohibits the neutrinoless double β decay.

The solution implies the relation $f_{e\mu} \gg f_{e\tau} \gg f_{\mu\tau}$ in the Zee model. The conservation of the new lepton number $L_{new} \equiv L_e - L_\mu - L_\tau$ requires $f_{\mu\tau} = 0$ means $m_{\mu\tau} = 0$. The realistic model is suggested in the extended Zee model with $L_{new} = -2$ doubly charged Higgs scalar field and 2-loop effects derive non-zero $m_{\mu\tau}$ [9]. Also the two loop effects in the original Zee model are studied[4].

Also the models which give the Zee type mass matrix are discussed in the viewpoint of R -parity violating SUSY model[10] or gauge mediated SUSY model[11]. Also three active and one sterile neutrino model is discussed in Ref.[12]. The phenomenology of h^+ is also analyzed[13].

The bi-maximal solution in Zee type mass matrix requires two degenerate heavy masses and one light neutrino. Solar neutrino oscillation experiment can resolve which mixing matrix is viable.

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